Mathematics and explication. Reply to Jean Paul van Bendegem

Kuipers, Theo A.F.

Published in:
EPRINTS-BOOK-TITLE

Publication date:
2005

Theo A.F. Kuipers

MATHEMATICS AND EXPLICATION
REPLY TO JEAN PAUL VAN BENDEGEM

Both specific claims of Jean Paul Van Bendegem are very plausible. First, there are many convincing mathematical arguments that are no genuine mathematical proofs and, second, the way in which these arguments build up our support of mathematical statements is quite similar to the way it is done in the empirical sciences. Since Van Bendegem is, like me, in general very much interested in the similarities between "science, philosophy, theology, and mathematics," I will start this reply by summarizing my view on the similarities and differences. Next I deal with his specific claims.

Mathematical Research as Concept Explication

Leaving theology here aside, I would like to claim that the basic similarity between philosophy and mathematics is the focus on the explication of informal concepts. In SiS I wrote (p. 8):

> For philosophy and mathematics the fourth type of program, the *explicative* research program, is the most important type. Such programs are directed at concept explication, i.e., the construction of a simple, precise and useful concept, which is, in addition, similar to a given informal concept (cf. Carnap 1963, pp. 1-18). For example, the concepts of 'logical consequence' and 'probability' have given rise to very successful explicative programs in the borderland between philosophy and mathematics. One of the main explicative programs dealt with in ICR is intended to explicate the intuitive idea of 'truthlikeness'. Although several analyses in the present book [SiS] could have been explicitly presented as examples of concept explication, we have made this identification in only a few chapters, and not even very rigorously at that, … .
>
> The strategy of concept explication is the following. From the intuitive concept to be explicated one tries to derive conditions of adequacy that the explicated concept will have to satisfy, and evident examples and counter-examples that the explicated concept has to include or exclude.

Let me also mention that explication may go further than the explication of informal concepts, it may also aim at the explication of intuitive judgments,

In: R. Festa, A. Aliseda and J. Peijnenburg (eds.), *Cognitive Structures in Scientific Inquiry* (*Poznań Studies in the Philosophy of the Sciences and the Humanities,* vol. 84), pp. 170-173. Amsterdam/New York, NY: Rodopi, 2005.

i.e., intuitions, including their justification, demystification or even undermining. A main example in ICR concerns the intuition about the functionality of choosing empirically more successful theories in order to enhance truth approximation. Another, certainly demystifying, example is the intuition that beauty may be an indication of the truth (Kuipers 2002). The strategy of "intuition explication" is a plausible extension of that involving concept explication.

So far the special nature of mathematical and philosophical research has been highlighted somewhat, but the similarity between concept explication and empirical research may not yet be very convincing. However, in several branches of mathematics the domain of research is quasi-empirical. For example, as Lakatos (1976) has demonstrated so beautifully, the history of the explication of the idea of a regular polyhedron and Euler's conjecture for all polyhedra that the number of vertices plus the number of faces equals the number of edges plus two, is such a quasi-empirical story. However, not all mathematical concepts and theorems have this feature. For example, the informal logico-mathematical notion of a group is primarily an abstract notion with, to be sure, many evident examples and non-examples. There is nevertheless quite some similarity between progress in explicative research on the one hand and (descriptive and explanatory) nomological and design research on the other. In SiS, pp. 263-4, I wrote:

> Like nomological research, this [explicative] task may be represented in terms of conceptual possibilities. Let us further assume first that there is a unique solution and hence a unique set of desired possibilities. Let a provisional explication also be conceived as (determining) a unique extension of conceptual possibilities. Then it is plausible to define formal progress in explicative research formally in the same way as in the case of nomological research.
>
> In real-life explicative research, however, the resemblance of a provisional explication has to be evaluated in other terms. In particular, such evaluation takes place in terms of evident examples, that is, evidently desired possibilities, evident 'non-examples', that is, evidently undesired possibilities, and, finally, so-called conditions of adequacy, that is, conditions to be fulfilled and which correspond to desired features. Hence, the definition of 'conceptual progress' in explicative research is straightforward. Provisional explication *Y* is better than provisional explication *X*, roughly speaking, if and only if *Y* treats more evident examples and non-examples properly and/or fulfills more conditions of adequacy.

The partial analogy between nomological and design research on the one hand and explicative research on the other is obvious, including the possibility of functionally equivalent explications. However, at least two differences with nomological research are very interesting. Whereas evident non-examples play an important role in explicative research, there is no nomological analogue for them, as this would require the realization of nomic impossibilities. Moreover, nomological research is more or less bound to a unique solution, whereas explicative research may well lead to the conclusion that two or more interesting explications can be given, which are functionally equivalent relative to the desired features but nevertheless mutually exclude each other.

Design research shares with explicative research this possibility of more than one useful solution. However, as in nomological research, there does not seem to be an analogue for evident non-examples in the case of design research. Moreover, it is argued in some detail in SiS, Chapter 9, that formal progress in design research is relatively easy to determine, but not so in concept explication. Hence, although there is also a strong analogy between design and explicative research, i.e., both aim at a certain product, the analogy is not perfect.

Let me use the opportunity to stress something that I forgot to do in SiS. Although I mention in SiS that explicative research can also pertain to crucial terms in the empirical sciences, I forgot to emphasize that in this case conditions of adequacy and evident examples and non-examples should agree as much as possible with up to date empirical, in particular nomological, research. As Hempel (1952, p. 12) already put it: "An explication of a given set of terms, then, combines essential aspects of meaning analysis and empirical analysis." Ideally, concept explication in the empirical sciences leads to an improved conceptual framework for further empirical research.

Mathematical Arguments

Van Bendegem's paper nicely illustrates that the transition from mathematics to philosophy of mathematics is methodologically not a big step. Its main aim is to start the explication of the informal notion of a mathematical argument, as a much weaker notion than that of a mathematical proof. As a matter of fact, the examples (b)-(g) are at least in part intended as evident cases of mathematical arguments not qualifying as proofs. Moreover, some of the general statements evidently function as a condition of adequacy or, very interestingly, as a "non-condition" of adequacy. In particular, the second claim in Section 4, according to which the combination of arguments for A and $A \supset B$ need not be an argument for B, is an intriguing and perhaps disputable but nevertheless clear illustration of an intended non-condition of adequacy. In that section, Van Bendegem also sets the stage for an explication of the way arguments change our degree of confidence in a mathematical statement.

Let me close with a question that intrigues me on the basis of reading Van Bendegem's paper: what are the differences and similarities between arguments that can be transformed into, or replaced by, genuine proofs and arguments which cannot? More specifically, I mean the following. As is wellknown, in many cases it is possible to prove a claim without really calculating and deducing the conclusion, but by giving a very elegant argument, of a mathematical and/or empirical nature, that immediately convinces everybody who starts to understand it. In Kuipers (1991), I collected 10 such examples. One of them deals with mixing white and red wine. Two identical bottles contain five glasses of wine, the one white, the other red. The bottles can contain six glasses. Now one pours a glassful from the bottle with red wine into the one with white wine, shakes the latter very well, and then pours a glass of the mixture back into the red wine bottle, and again shakes very well. Which bottle has the highest concentration of wine that originally comes from the other bottle? Of course, one can make a calculation, leading to the conclusion that the concentrations are the same. However, one may also immediately see this solution by realizing that otherwise the total amount of white and red wine would have changed. Assuming that this in itself cannot count as a proof but can be transformed into an indirect proof, it seems to be an argument of a third kind: it is neither a proof in the strict sense nor an argument that merely increases our degree of conviction.

REFERENCES

Carnap, R. (1963). *Logical Foundations of Probability*. Chicago: The University of Chicago Press.

Hempel, C. (1952). *Fundamentals of Concept Formation in Empirical Science*. Chicago: The University of Chicago Press.

Kuipers, T. (1991). Dat Vind Ik Nou Mooi. In: R. Segers (ed.), *Visies op Cultuur en Literatuur. Opstellen naar Aanleiding van het Werk van J.J.A. Mooij*, pp. 69-75. Amsterdam/Atlanta: Rodopi.

Kuipers, T. (2002). Beauty, a Road to The Truth. *Synthese* **131** (3), 291-328.

Lakatos, I. (1976). *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge: Cambridge University Press.